

Table 25.2

COEFFICIENTS FOR DIFFERENTIATION

$$\text{Differentiation Formula: } \frac{df(x)}{dx} \bigg|_{x=x_j} \approx \frac{k!}{m!h^k} \sum_{i=0}^m A_i f(x_i)$$

FIRST DERIVATIVE (k=1)								THIRD DERIVATIVE (k=3)								
* j	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	$\frac{h^k}{k!}$ Error	j	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	$\frac{h^k}{k!}$ Error	*
Three Point (m=2)								Four Point (m=3)								
0	-3	4	-1				$\frac{1}{3} h^3 f^{(3)}$	0	-1	3	-3	1			$-\frac{1}{4} h^4 f^{(4)}$	
1	-1	0	1				$-\frac{1}{6} h^3 f^{(3)}$	1	-1	3	-3	1			$-\frac{1}{12} h^4 f^{(4)}$	
2	1	-4	3				$\frac{1}{3} h^3 f^{(3)}$	2	-1	3	-3	1			$\frac{1}{12} h^4 f^{(4)}$	
								3	-1	3	-3	1			$\frac{1}{4} h^4 f^{(4)}$	
Four Point (m=3)								Five Point (m=4)								
0	-11	18	-9	2			$-\frac{1}{4} h^4 f^{(4)}$	0	-10	36	-48	28	-6		$\frac{7}{24} h^5 f^{(5)}$	
1	-2	-3	6	-1			$-\frac{1}{12} h^4 f^{(4)}$	1	-6	20	-24	12	-2		$\frac{1}{24} h^5 f^{(5)}$	
2	1	-6	3	2			$-\frac{1}{12} h^4 f^{(4)}$	2	-2	4	0	-4	2		$-\frac{1}{24} h^5 f^{(5)}$	
3	-2	9	-18	11			$\frac{1}{4} h^4 f^{(4)}$	3	2	-12	24	-20	6		$\frac{1}{24} h^5 f^{(5)}$	
								4	6	-28	48	-36	10		$\frac{7}{24} h^5 f^{(5)}$	
Five Point (m=4)								Six Point (m=5)								
0	-50	96	-72	32	-6		$\frac{1}{5} h^5 f^{(5)}$	0	-85	355	-590	490	-205	35	$-\frac{5}{16} h^6 f^{(6)}$	
1	-6	-20	36	-12	2		$-\frac{1}{20} h^5 f^{(5)}$	1	-35	125	-170	110	-35	5	$-\frac{1}{48} h^6 f^{(6)}$	
2	2	-16	0	16	-2		$-\frac{1}{20} h^5 f^{(5)}$	2	-5	-5	50	-70	35	-5	$\frac{1}{48} h^6 f^{(6)}$	
3	-2	12	-36	20	6		$-\frac{1}{20} h^5 f^{(5)}$	3	5	-35	70	-50	5	5	$-\frac{1}{48} h^6 f^{(6)}$	
4	6	-32	72	-96	50		$\frac{1}{5} h^5 f^{(5)}$	4	-5	35	-110	170	-125	35	$\frac{1}{48} h^6 f^{(6)}$	
								5	-35	205	-490	590	-355	85	$\frac{5}{16} h^6 f^{(6)}$	
Six Point (m=5)								FOURTH DERIVATIVE (k=4)								
0	-274	600	-600	400	-150	24	$-\frac{1}{6} h^6 f^{(6)}$	j	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	$\frac{h^k}{k!}$ Error	*
1	-24	-130	240	-120	40	-6	$\frac{1}{30} h^6 f^{(6)}$	Five Point (m=4)								
2	6	-60	-40	120	-30	4	$-\frac{1}{60} h^6 f^{(6)}$	0	1	-4	6	-4	1		$-\frac{1}{12} h^5 f^{(5)}$	
3	-4	30	-120	40	60	-6	$\frac{1}{60} h^6 f^{(6)}$	1	1	-4	6	-4	1		$-\frac{1}{24} h^5 f^{(5)}$	
4	6	-40	120	-240	130	24	$-\frac{1}{30} h^6 f^{(6)}$	2	1	-4	6	-4	1		$-\frac{1}{144} h^6 f^{(6)}$	
5	-24	150	-400	600	-600	274	$\frac{1}{6} h^6 f^{(6)}$	3	1	-4	6	-4	1		$\frac{1}{24} h^5 f^{(5)}$	
								4	1	-4	6	-4	1		$\frac{1}{12} h^5 f^{(5)}$	
SECOND DERIVATIVE (k=2)								Six Point (m=5)								
* j	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	$\frac{h^k}{k!}$ Error	0	15	-70	130	-120	55	-10	$\frac{17}{144} h^6 f^{(6)}$	
Three Point (m=2)								1	10	-45	80	-70	30	-5	$\frac{5}{144} h^6 f^{(6)}$	
0	1	-2	1				$-\frac{1}{2} h^3 f^{(3)}$	2	5	-20	30	-20	5	0	$-\frac{1}{144} h^6 f^{(6)}$	
1	1	-2	1				$-\frac{1}{24} h^4 f^{(4)}$	3	0	5	-20	30	-20	5	$-\frac{1}{144} h^6 f^{(6)}$	
2	1	-2	1				$\frac{1}{2} h^3 f^{(3)}$	4	-5	30	-70	80	-45	10	$\frac{5}{144} h^6 f^{(6)}$	
								5	-10	55	-120	130	-70	15	$\frac{17}{144} h^6 f^{(6)}$	
Four Point (m=3)								FIFTH DERIVATIVE (k=5)								
0	6	-15	12	-3			$\frac{11}{24} h^4 f^{(4)}$	j	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	$\frac{h^k}{k!}$ Error	*
1	3	-6	3	0			$-\frac{1}{24} h^4 f^{(4)}$	Six Point (m=5)								
2	0	3	-6	3			$-\frac{1}{24} h^4 f^{(4)}$	0	-1	5	-10	10	-5	1	$-\frac{1}{48} h^6 f^{(6)}$	
3	-3	12	-15	6			$\frac{11}{24} h^4 f^{(4)}$	1	-1	5	-10	10	-5	1	$-\frac{1}{80} h^6 f^{(6)}$	
								2	-1	5	-10	10	-5	1	$-\frac{1}{240} h^6 f^{(6)}$	
Five Point (m=4)								3	-1	5	-10	10	-5	1	$\frac{1}{240} h^6 f^{(6)}$	
0	35	-104	114	-56	11		$-\frac{5}{12} h^5 f^{(5)}$	4	-1	5	-10	10	-5	1	$\frac{1}{80} h^6 f^{(6)}$	
1	11	-20	6	4	-1		$\frac{1}{24} h^5 f^{(5)}$	5	-1	5	-10	10	-5	1	$\frac{1}{48} h^6 f^{(6)}$	
2	-1	16	-30	16	-1		$\frac{1}{180} h^6 f^{(6)}$									
3	-1	4	6	-20	11		$-\frac{1}{24} h^5 f^{(5)}$									
4	11	-56	114	-104	35		$\frac{5}{12} h^5 f^{(5)}$									
Six Point (m=5)																
0	225	-770	1070	-780	305	-50	$\frac{137}{360} h^6 f^{(6)}$	0	-1	5	-10	10	-5	1	$-\frac{1}{48} h^6 f^{(6)}$	
1	50	-75	-20	70	-30	5	$-\frac{13}{360} h^6 f^{(6)}$	1	-1	5	-10	10	-5	1	$-\frac{1}{80} h^6 f^{(6)}$	
2	-5	80	-150	80	-5	0	$\frac{1}{180} h^6 f^{(6)}$	2	-1	5	-10	10	-5	1	$-\frac{1}{240} h^6 f^{(6)}$	
3	0	-5	80	-150	80	-5	$\frac{1}{180} h^6 f^{(6)}$	3	-1	5	-10	10	-5	1	$\frac{1}{240} h^6 f^{(6)}$	
4	5	-30	70	-20	-75	50	$-\frac{13}{360} h^6 f^{(6)}$	4	-1	5	-10	10	-5	1	$\frac{1}{80} h^6 f^{(6)}$	
5	-50	305	-780	1070	-770	225	$\frac{137}{360} h^6 f^{(6)}$	5	-1	5	-10	10	-5	1	$\frac{1}{48} h^6 f^{(6)}$	

Compiled from W. G. Bickley, Formulae for numerical differentiation, Math. Gaz. 25, 19-27, 1941 (with permission).